Fault detection and isolation of faults in a multivariate process with Bayesian network

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The main objective of this paper is to present a new method of detection and isolation with a Bayesian network. For that, a combination of two original works is made. The first one is the work of Li et al. [1] who proposed a causal decomposition of the T2 statistic. The second one is a previous work on the detection of fault with Bayesian networks [2], notably on the modeling of multivariate control charts in a Bayesian network. Thus, in the context of multivariate processes, we propose an original network structure allowing to decide if a fault has appeared in the process. This structure permits the isolation of the variables implicated in the fault. A particular interest of the method is the fact that the detection and the isolation can be made with a unique tool: a Bayesian network.

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1. Introduction
Nowadays, monitoring of complex manufacturing systems is becoming an essential task in order to: insure a safe production (for humans and materials), reduce the variability of products or reduce manufacturing cost. Classically, in the literature, three approaches can be found for the process monitoring [3,4]: the knowledge-based approach, the model-based approach and the data-driven approach. The knowledge-based category represents methods based on qualitative models: Digraphs; Fault Trees [5]; Case Based Reasoning [6]. The model-based approach is based on analytical (physical) models able to simulate the system [7]. Though, at each instant, the theoretical value of each sensor can be known for the normal operating state of the system. As a consequence, it is relatively easy to see if the real process values are similar to the theoretical values. However, the major drawback of this family of techniques is that a detailed model of the process is required in order to monitor it efficiently. An effective detailed model can be very difficult, time consuming and expensive to obtain, particularly for large-scale systems with many variables. The data-driven approaches are a family of different techniques based on the analysis of the real data extracted from the process [8]. These methods are based on rigorous statistical developments of the process data (i.e., control charts, methods based on Principal Component Analysis, Projection to Latent Structure or Discriminant Analysis) [3]. Since we are monitoring large multivariate processes, we will work in the data-driven monitoring framework.

To achieve this activity of data-driven monitoring, some authors call this AEM (Abnormal Event Management) [4]. This is composed of three principal steps: firstly, a timely detection of an abnormal event; secondly, diagnosing its causal origins (or root causes); and finally, taking appropriate decisions and actions to return the process in a normal working state. As the third step is specific to each process, literature generally focuses on the two first steps: fault detection and diagnosis, named FDD [9]. We will call “fault” an abnormal event (like an excessive pressure in a reactor, or a low quality of a part of a product, and so on), usually defined as a departure from an acceptable range of an observed variable or a calculated parameter of the process [4]. Generally, a monitoring technique is dedicated to one specific step: detection or diagnosis. In the literature, one can find many data-driven techniques for the fault detection: univariate statistical process control (Shewhart charts) [10,11], multivariate statistical process control (T2 and Q charts) [12,13], and some PCA (Principal Component Analysis) based techniques [14] like Multiway PCA or Moving PCA [15] used for the detection step. Kano et al. [16] make comparisons between these different techniques.

An efficient fault detection and isolation tool should be able to isolate the variables implicated in the fault, in order to help the process operator to identify the root cause (the physical cause) of the fault. Some methods exist to solve this problem (see Section 2.2), which are based on a decomposition of the T2 statistic. But, each of these methods uses different tools for the fault detection and the fault isolation (variables implicated in the fault), like control
charts, statistical decompositions, Bayesian networks, etc. From a practical point of view, it would be more interesting to combine the main advantages of these techniques and to exploit them jointly in one single tool. Recently, the application of Bayesian networks for the fault detection and diagnosis has been used with success, in the data-driven context [17,18], but also in the model-based context [19]. The objective of this article is to propose an improvement of the decomposition method of Li et al. [1], in order to use a sole Bayesian network to detect a fault and to isolate the implicated variables in this fault.

The article is structured as follows: Section 2 presents preliminaries needed for a correct understanding of the article; Section 2.1 highlights some aspects of Bayesian networks; Section 2.2 presents the various $T^2$ decompositions (causal and MYT); in Section 3 we show how to construct some multivariate control charts with a Bayesian network and how to exploit the network in order to isolate the detected faults; two examples of the approach are presented in Section 4; finally, in the last section, we conclude on the proposed approach.

2. Preliminaries

2.1. Bayesian networks

2.1.1. Definition

A Bayesian network (BN) [20,21] is an acyclic graph where each variable is a node (that can be continuous or discrete). Edges of the graph represent dependence between linked nodes. A formal definition is given here:

A Bayesian network is a triplet $\{G,E,D\}$ where:

- $G$ is a directed acyclic graph, $G=\langle V,A \rangle$, with $V$ the set of nodes of $G$, and $A$ the set of edges of $G$;
- $E$ is a finite probabilistic space $\Omega \times Z \times p$, with $\Omega$ a non-empty space, $Z$ a collection of subspaces of $\Omega$, and $p$ a probability measure on $Z$ with $p(\Omega) = 1$;
- $D$ is a set of random variables associated to the nodes of $G$ and defined on $E$ such as:

$$p(V_1, V_2, \ldots, V_n) = \prod_{i=1}^{n} p(V_i | C(V_i)) \quad (1)$$

with $C(V_i)$ the set of causes (parents) of $V_i$ in the graph $G$.

2.1.2. Dependences in Bayesian network

Theoretically, variables $X_1, X_2, \ldots, X_n$ can be discrete or continuous. But, in practice, for exact computation, only the discrete and the Gaussian case can be treated (see Ref. [22]). Such a network is often called Conditional Gaussian Network (CGN). In this context, to ensure availability of exact computation methods, discrete variables are not allowed to have continuous parents [23,24].

Practically, the conditional probability distribution is described for each node by his Conditional Probability Table (CPT) (see Ref. [22]). In a CGN, three cases of CPT can be found. The first one is for a discrete variable with discrete parents. By example, we take the case of two discrete variables $A$ and $B$ of respective dimensions $a$ and $b$ (with $a_1, a_2, \ldots, a_b$ the different modalities of $A$, and $b_1, b_2, \ldots, b_b$ the different modalities of $B$). If $A$ is parent of $B$, then the CPT of $B$ is represented in Table 1.

We can see that the utility of the CPT is to resume the information about the relation of $B$ with its parent. We can denote that the dimension of this CPT is $a \times b$. In general, the dimension of the CPT of a discrete node (dimension $x$) with $p$ parents (discrete) $Y_1, Y_2, \ldots, Y_p$ (dimension $y_1, y_2, \ldots, y_p$) is $x \prod_{i=1}^{p} y_i$.

The second case of CPT is for a continuous variable with discrete parents. Assuming that $B$ is a Gaussian variable, and that $A$ is a discrete parent of $B$ with a modalities, the CPT of $B$ can be represented as in Table 2 where $P(B(a_1) \sim N(\mu_{a_1}, \Sigma_{a_1}))$ indicates that $B$ conditioned to $A=a_1$ follows a multivariate normal density function with parameters $\mu_{a_1}$ and $\Sigma_{a_1}$. If we have more than one discrete parent, the CPT of $B$ will be composed of $\prod_{i=1}^{p} y_i$ Gaussian distributions where $y_i$ represents the respective number of modalities of the parent nodes $Y_1, Y_2, \ldots, Y_p$.

The third case is when a continuous node $B$ has a continuous parent $A$. In this case, we obtain a linear regression and we can write, for a fixed value $a$ of $A$, that $B$ follows a Gaussian distribution $P(B(a) \sim N(\mu_B + \beta \times a; \Sigma_B))$ where $\beta$ is the regression coefficient. Evidently, the three different cases of CPT enumerated can be combined for different cases where a continuous variable has several discrete parents and several continuous (Gaussian) parents.

The classical usage of a Bayesian network (or Conditional Gaussian Network) is to enter evidence in the network (an evidence is the observation of the values of a set of variables). Thus, the information given by the evidence is propagated in the network in order to update the knowledge and obtain a posteriori probabilities on the non-observed variables. This propagation mechanism is called inference. As its name suggests, in a Bayesian network, the inference is based on the Bayes rule. Many inference algorithms (exact or approximate) have been developed, but one of the more exploited is the junction tree algorithm [25].

2.2. Existental methods for isolation

2.2.1. The MYT decomposition

As we previously stated, a method for the fault detection in multivariate processes is the $T^2$ control chart. However, this chart does not give any information about the diagnosis of the out-of-control situation. For that, many techniques have been proposed in the literature [26,27]. An interesting decomposition of the $T^2$ has been proposed by Mason et al. [26], namely MYT decomposition. The authors have proved that several isolation methods can be considered like special cases of MYT decomposition [26]. The principle of this method is to decompose the $T^2$ statistic in a limited number of orthogonal components which are also statistical distances. This decomposition is the following:

$$T^2 = T^2_1 + T^2_2 + \ldots + T^2_{p-1} = T^2_{1,2} + T^2_{3,1,2} + T^2_{4,1,2,3} + \cdots + T^2_{p,1,2,3,\ldots,p-1} \quad (2)$$

\begin{table}[h]
\centering
\caption{CPT of a discrete node with discrete parents.}
\begin{tabular}{ccc}
\hline
$A$ & $B$ \\
\hline
& $b_1$ & $b_2$ & \ldots & $b_b$ \\
$a_1$ & $P(b_1|a_1)$ & $P(b_2|a_1)$ & \ldots & $P(b_b|a_1)$ \\
$a_2$ & $P(b_1|a_2)$ & $P(b_2|a_2)$ & \ldots & $P(b_b|a_2)$ \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
$a_n$ & $P(b_1|a_n)$ & $P(b_2|a_n)$ & \ldots & $P(b_b|a_n)$ \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{CPT of a Gaussian node with discrete parents.}
\begin{tabular}{cc}
\hline
$A$ & $B$ \\
\hline
$a_1$ & $P(B(a_1) \sim N(\mu_{a_1}, \Sigma_{a_1}))$ \\
$a_2$ & $P(B(a_2) \sim N(\mu_{a_2}, \Sigma_{a_2}))$ \\
\vdots & \vdots \\
$a_n$ & $P(B(a_n) \sim N(\mu_{a_n}, \Sigma_{a_n}))$ \\
\hline
\end{tabular}
\end{table}
where $T^2_{J,k}$ represents the $T^2$ statistic of the regression of the variables $X_j$ and $X_k$ on the variable $X_i$. We remark that there exists a large number of different decompositions ($p!$) and so, there exists a large number of different terms ($p \times 2^{p-1}$). We illustrate this by using a process with three variables, decompositions available are:

\[
\begin{align*}
T^2 &= T^2_1 + T^2_{2,1} + T^2_{3,1,2} \\
T^2 &= T^2_1 + T^2_{2,1} + T^2_{3,1,3} \\
T^2 &= T^2_2 + T^2_{2,2} + T^2_{2,3,1,2} \\
T^2 &= T^2_2 + T^2_{2,2} + T^2_{2,3,1,3} \\
T^2 &= T^2_2 + T^2_{1,3} + T^2_{2,1,3} \\
T^2 &= T^2_2 + T^2_{1,3} + T^2_{2,2,3} \\
T^2 &= T^2_2 + T^2_{2,3} + T^2_{2,3,1,3}
\end{align*}
\]

The computation of the different terms is not detailed in this paper, but we refer the reader to Mason et al. [26]. The terms $T^2_i$ are called non-conditional terms and that the other terms are called conditional terms. Each terms follows a Fisher distribution law:

\[
T^2_{J=1,\ldots,j} = \frac{(m + 1)(m - 1)}{m(m - k - 1)} F_{1,m-k-1}
\]

where $k$ is the number of conditioned factors. For the non-conditional terms ($k=0$), the equation is simplified to:

\[
T^2_i = \frac{m + 1}{m} F_{1,m-1}
\]

The monitoring allows the detection of a problem on each term of the decomposition. For example, if the term $T^2_{J,2}$ is responsible of the out-of-control of the process, one can immediately search a root cause on a tuning of the correlation between these two variables. However, for a more simple computation, one can use a $T^2$ control chart for the detection of the fault, and in the case of a detected fault the MYT decomposition is applied. The analysis of the different terms is made in levels order: (firstly $T^2_1$, then $T^2_2$, and finally terms $T^2_{i,3}$, $T^2_{i,1,2}$, $T^2_{i,1,3}$, $T^2_{i,2,3}$) until that one finds the term responsible of the detection on the $T^2$ control chart.

The main advantage of the MYT decomposition is that the diagnosis is done without any samples of previous faults. Another advantage is that this method is based on the same statistic tool that the $T^2$ control chart.

### 2.2.2. The causation-based $T^2$ decomposition

The MYT method is very interesting, but it has a major drawback: the number of term to compute is large ($p \times 2^{p-1}$). For example, for a process with 20 variables, more than 10 millions of terms are needed. A five steps algorithm has been proposed by Mason et al. [28] in order to reduce the number of terms to compute. However, Li et al. [1] stated that the number of terms is always too large. Therefore, they propose a new method using a Bayesian network: the causation-based $T^2$ decomposition. A causal graph of the process allows the reduction of the number of terms to only $p$ increases the performances of the diagnosis. The basic assumption of the method proposed by Li et al. [1] is that the process can be modeled with a causal Bayesian network where each variable of the process is a Gaussian univariate variable. If a Bayesian network represents solely some Gaussian continuous variables, it is also called linear Gaussian model. For a process with 3 variables, we can obtain, for example, the network in Fig. 1.

Concerning the modeling of the process by a linear Gaussian model, the authors make the distinction between two types of decomposition MYT: “for a given decomposition of the $T^2$, if it exists a term $T^2_{1,\ldots,i}$ such the ensemble of the variable $\{X_1, \ldots, X_i\}$ includes at least one descendant of $X_i$, then this decomposition is a type A decomposition, if not, the decomposition is a type B decomposition”. So, we can sort as shown in Table 3 the different decompositions of the process with 3 variables given in Fig. 1.

Li et al. [1] prove, based on Hawkins [27], that the type A decompositions allow a less accurate diagnosis than the type B decompositions. Moreover, they proved that in the context of linear Gaussian models, all the type B decompositions converge to a sole decomposition that the authors named “causation-based $T^2$ decomposition”. Indeed, each type B decomposition converges to the causal decomposition of the $T^2$ given in Eq. (6), where $PA(X_i)$ represents the parents of the variable $X_i$ in the causal graph:

\[
T^2 = \sum_{i=1}^{p} T^2_{PA(X_i)}
\]

Thereby, the causation-based $T^2$ decomposition of the example of Fig. 1 is the following: $T^2 = T^2_1 + T^2_{2,1} + T^2_{3,1}$.

The authors have proposed a procedure of detection and isolation of fault using the new causal decomposition. Firstly, a linear Gaussian Bayesian network is constructed in order to represent the different causal relations between the process variables. Secondly, the process is monitored with a $T^2$ control chart. In the case of an out-of-control situation, the $T^2$ is decomposed by the causation-based $T^2$ decomposition of Eq. (6). In this equation, each $T^2_{PA(X_i)}$ is independent and, in the case of known parameters, follows a $\chi^2$ distribution law with one degree of freedom. So, each $T^2_{PA(X_i)}$ can be compared to the limit $\chi^2_{1,\text{alpha}}$, which is the quantile at the value $\alpha$ ($\alpha$ is the false alarm rate) of the $\chi^2$ distribution with one degree of freedom. A significant term $T^2_{PA(X_i)}$ (higher than the limit) signifies that the variable $X_i$ has been implicated in the fault. Fig. 2 represents the process monitoring diagram with the given method.

The approach developed by Li et al. [1] exploits some threshold given by quantiles of statistical laws. This method considerably increases the performances compared to the MYT method, with less computation resource. However, we can notice in Fig. 2 that several tools are used for this technique: control chart, Bayesian network, statistical computations. We propose, in the next section, that the monitoring of a multivariate process by the method of the causal-based $T^2$ decomposition can be entirely made using Bayesian network alone.

<table>
<thead>
<tr>
<th>Decomposition</th>
<th>Type</th>
</tr>
</thead>
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<tr>
<td>$T^2 = T^2_1 + T^2_{2,1} + T^2_{3,1,2}$</td>
<td>Type B</td>
</tr>
<tr>
<td>$T^2 = T^2_1 + T^2_{2,1} + T^2_{3,1,3}$</td>
<td>Type B</td>
</tr>
<tr>
<td>$T^2 = T^2_2 + T^2_{2,2} + T^2_{3,1,2}$</td>
<td>Type A</td>
</tr>
<tr>
<td>$T^2 = T^2_2 + T^2_{2,2} + T^2_{2,3,1,3}$</td>
<td>Type A</td>
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<tr>
<td>$T^2 = T^2_2 + T^2_{1,3} + T^2_{2,1,3}$</td>
<td>Type A</td>
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<td>$T^2 = T^2_2 + T^2_{1,3} + T^2_{2,2,3}$</td>
<td>Type A</td>
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<tr>
<td>$T^2 = T^2_2 + T^2_{2,3} + T^2_{2,3,1,3}$</td>
<td>Type A</td>
</tr>
</tbody>
</table>

### Table 3

Decomposition types for the three variables process.
3. The proposed approach

3.1. Control charts with Bayesian network

In previous work [2], we have demonstrated that a $T^2$ control chart [12] could be modeled with a Bayesian network. For that, we use two nodes: a Gaussian multivariate node $X$ representing the data and a bimodal node $E$ representing the state of the process. The bimodal node $E$ has the following modalities: IC for “in-control” and OC for “out-of-control”. Assuming that $\mu$ and $\Sigma$ are respectively the mean vector and the variance–covariance matrix of the process, we can monitor the process with the following rule: if $P(E|X) < P(E)$ then the process is out-of-control. This Bayesian network is represented in Fig. 3 where the conditional probability tables of each node are given.

In Fig. 3, we can observe that a coefficient $c$ is implicated in the modeling of the control chart by Bayesian network. This coefficient is the root (different of 1) of the following equation:

$$1 - c + \frac{p c}{CL} \ln(c) = 0$$

(7)

where $p$ is the dimension (number of variables) of the system to monitor, and $CL$ is the control limit of the equivalent $T^2$ control chart. The demonstration of the computation of $c$ is given in Appendix A. In numerous cases, $CL$ is equal to $\chi^2_{p, \alpha}$, the quantile at the value $\alpha$ of the distribution of the $\chi^2$ with $p$ degrees of freedom [11]. $\alpha$ allows us to tune the false alarm rate of the control chart.

3.2. Improvement of the causal decomposition

The method proposed by Li et al. [1] uses a Bayesian network to select the different terms of the MYT decomposition to compute. For the computation of the different terms of the causation-based $T^2$ decomposition, and for the associated decisions (use of the threshold), the authors do not use the Bayesian network in an optimal way. Indeed, they use a $T^2$ control chart and compute each $T^2_{PA(X_i)}$ out of the network. We will prove that it is possible to make all the computations in the network.

We propose an extension to Li et al. [1] by the computation of the different $T^2_{PA(X_i)}$ and the decisions associated with each one. The fault isolation with the causation-based $T^2$ decomposition, like the MYT decomposition, is a monitoring of regressed variables, with the use of univariate control charts. In the previous section, we have demonstrated how to model, in a Bayesian network, a multivariate control chart like the $T^2$ control chart. But, a univariate control chart like a Shewhart control chart is simply a particular case of a multivariate control chart like the $T^2$ control chart. Indeed, the computation of the $T^2$ is given by:

$$T^2 = (x - \mu)^T \Sigma^{-1} (x - \mu)$$

(8)

In the univariate case, $x = x$, $\mu = \mu$ and $\Sigma = \sigma^2$, and the previous equation becomes:

$$T^2 = \frac{(x - \mu)^2}{\sigma^2}$$

(9)

In this univariate case, the $T^2$ statistic follows a $\chi^2$ distribution law with one degree of freedom. So, it is possible to improve the method developed by Li et al. [1]. We propose to monitor directly the different values of the $T^2_{PA(X_i)}$ in the Bayesian network. For that, we add a bimodal variable to each univariate node of the Bayesian network. If we have a graph representing a system with three variables (see Fig. 1), we then obtain a network with six nodes: 3 continuous (Gaussian univariate) and 3 discrete (bimodal).

The discrete nodes added to the initial structure of the network allow direct isolation of the responsible variables in an out-of-control situation. These nodes model a control chart like the $T^2$ control chart. Indeed, the computation of the $T^2$ is given in Appendix A. In numerous cases, $T^2_{PA(X_i)}$ is equal to $\chi^2_{p, \alpha}$, the quantile at the value $\alpha$ of the distribution of the $\chi^2$ with $p$ degrees of freedom [11]. $\alpha$ allows us to tune the false alarm rate of the control chart.
the implicated variables (isolation). Fig. 5 presents the form of the Bayesian network for the process with 3 variables. This network is able to perform all the steps of the Li et al. method (Fig. 2).

4. Applications

In order to better understand the new proposed method, two studies are proposed: a hot forming process and the well known Tennessee Eastman Process.

4.1. Case study I: the hot forming process

4.1.1. Process description

The hot forming process (Fig. 6) has 5 variables: one quality variable (X5: final dimension of workpiece) and four process variables (X1, temperature; X2, material flow stress; X3, tension in workpiece; and X4, blank holding force (BHF)). Because the material flow stress (X2) and the tension in a workpiece (X3) directly affect the final workpiece dimension (X5) – where “directly” means that the causal influences are not mediated through other variables – X2 and X3 are connected to X5 by directed arcs. The BHF (X4) also affects the dimension of the workpiece (X5), but only indirectly, i.e., through the tension in the workpiece (X3). Thus, there is no directed arc between X4 and X5. Similar interpretations can be applied to the causal relationships between other variables. From the causal network, we construct the Bayesian network shown in Fig. 7.

4.1.2. Simulations and results

Because this system has five variables, there are five potential single-fault scenarios, each with only one variable having a mean shift and 26 potential multiple-fault scenarios, each with more than one variable having mean shifts. A total of 31 fault scenarios are shown in Table 4 here δ denotes the magnitude of the mean shift in the unit of standard deviation.

<table>
<thead>
<tr>
<th>No.</th>
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<th>Detection rate</th>
<th>Diagnosis rate</th>
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<td></td>
<td></td>
<td></td>
<td>BN</td>
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<tr>
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<td>87.68</td>
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<td>42.68</td>
<td>10.72</td>
</tr>
<tr>
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<td>δ δ δ δ δ</td>
<td>99.96</td>
<td>53.46</td>
<td>3.82</td>
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<td>δ δ δ δ δ</td>
<td>100.00</td>
<td>78.66</td>
<td>43.24</td>
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<td>60.18</td>
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<td>55.74</td>
<td>63.81</td>
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<td>δ δ δ δ δ</td>
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<td>100.00</td>
<td>35.72</td>
<td>45.64</td>
</tr>
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</table>

Mean 96.13 61.77 32.88
For a given scenario, we have simulated 1000 normal (free-fault) observations in order to learn the parameters of the network. Then, we simulated 5000 observations of the scenario with a shift magnitude of 3 ($\delta = 3$). The 5000 observations are given to the Bayesian network. The Bayesian network evaluates the state of the process (computes the probabilities of in-control and out-of-control) and gives the most probable variables responsible for an out-of-control situation. We have taken a global false alarm rate of 5% (for detection), and local false alarm rate of 1% (for isolation). For each discrete node, a priori probability has been set to 0.5. So, for each discrete variable, a probability to be out-of-control greater than 0.5 signifies that an out-of-control situation is detected or identified. For each scenario, the detection rate and the diagnosis rate have been computed for the Bayesian network approach and for the MYT approach (see Table 4). The detection rate is the number of detected
observation of the 5000 out-of-control observations, expressed in percent. The diagnosis rate is the number of correctly diagnosed observations of the detected observation, also expressed in percent.

In Table 4 for each multiple-fault scenarios (6–31) one can easily see that the detection rate is good. Concerning the single-fault scenarios (1–5), the detection rate is lower than for the multiple-fault ones. Indeed, as the step is only on one variable, the global step has a lower magnitude than for the multiple-fault ones, and so is less detectable. An interesting remark is that the more the fault is on an initial variable and the less it is detectable. For example, the step on variable X1 (first in the causal graph) is only of 62.58%, the same step on variable X3 is 82.92% and the same step on variable X5 reaches 96.24%. Indeed, more the fault affects a root causal variable (first in the causal graph) and more this fault is diluted and so becomes non-detectable.

Concerning the diagnosis or isolation, we can see that more variables are implicated and more the diagnosis is difficult (inverse conclusion of the detection). But, like for the detection, we can see that more the implicated variables are root causal variables, and more the diagnosis is difficult. For example, the diagnosis is better for a step on X2 and X5 (scenario 12, good diagnosis of 89.41%) than for a step on X4 (a more causal root variable than X2) and X5 (scenario 15, good diagnosis of 64%). These different analyses are presented in Table 5.

For a better understanding of the proposed method and particularly of the proposed Bayesian network, we give results of the scenario 8 (steps on X1 and X4) for the first 50 observations. These results are presented graphically in Fig. 8. In order to see the difference, we firstly give 50 observations of in-control situation. Moreover, we also add classical fault detection and isolation tools on the left part of the figure: the first one is the $T^2$ control chart, and the five above are the classical (Shewhart) control chart of the different variables. On the right of the figure, we present the probabilities given by the network in Fig. 7. The first graph represents the probability of the process to be out-of-control. As this probability is higher than 0.5, the process is out-of-control. The conclusions (detections) are equivalent to the $T^2$ control chart ones. The other five graphs represent the probability that the variable is implicated in the fault. Like the first graph, if the probability is greater than 0.5, it signifies that the variable can be considered as a root cause of the fault.

In Fig. 8, we can see that if we use a classical approach, we can conclude that all the variables are causes of the detected fault. But, with the Bayesian network method, we can see easily that only the variables X1 and X4 are root causes of the fault (good diagnosis for scenario 8).

### 4.1.3. Magnitude influence

In order to study the performances of the Bayesian network approach, the magnitude step has been changed. In Fig. 9, the mean of the diagnosis rate for the Bayesian network approach and for the MYT approach are represented, function of the magnitude step. One can notice that more the step magnitude is low and more the diagnosis is non-efficient. However, in each case, the diagnosis rate of the Bayesian network approach is better than the diagnosis rate of the MYT.

### 4.2. Case study II: the Tennessee Eastman Process

#### 4.2.1. Presentation of the TEP

We have tested our approach on the Tennessee Eastman Process (Fig. 10). The Tennessee Eastman Process (TEP) is a chemical process. It is not a real process but the simulation of a process created by the Eastman Chemical Company to provide a realistic industrial process in order to evaluate process control and monitoring methods. The article of Ref. [29] entirely describes this process. It is composed of five major operation units: a reactor, a condenser, a compressor, a stripper and a separator. Four gaseous reactants A, C, D, E and an inert one B are fed to the reactor where the liquid products F, G and H are formed. This process has 12 input variables and 41 output variables. The TEP has 20 types of identified faults. This process is a benchmark problem for control techniques because it is open-loop unstable. Many articles present the TEP and test monitoring approaches on it. We can cite [16] or [30–32] with the plant-wide control structure recommended in Ref. [33].

#### 4.2.2. Fault F4 case

In this section the particular case of the fault F4 is studied. This fault is a step in the reactor cooling water inlet temperature. In the TEP, no sensor is dedicated to this temperature. In the TEP, no sensor is dedicated to this temperature. However, this
fault can be observed by the values of two other variables $X_9$ and $X_{51}$, respectively the reactor temperature (in $^\circ$C) and the reactor cooling water flow (in m$^3$ h$^{-1}$). Fig. 11 gives comparisons of variables $X_9$ and $X_{51}$ in the fault free case and in the F4 case. On the graphs (c) and (d), the fault F4 is introduced from the observation 161. Fault F4 has the effect of increasing the value of the temperature in the reactor (variable $X_9$). At this instant, the control structure [33] increases the flow of the reactor cooling water (variable $X_{51}$). For that reason, $X_9$ decreases rapidly to the normal temperature.

4.2.3. BN results

In a first step, 800 observations of the fault free case have been exploited to construct the causal Bayesian network of the process. The PC algorithm [34] has been used with the Fisher’s z-transform for the estimation of partial correlations allowing the evaluation of the conditional independences [35]. The causal network found with the algorithm is represented in Fig. 12.

The Bayesian network approach has been tested on the fault F4 of the TEP. 800 observations of F4 have been presented to the Bayesian network. The network has detected all the 800 faulty
observations, and correctly diagnosed (variables 9 and 51 are responsible of the fault) 771 observations (96.38%). Therefore, the application of the Bayesian network approach to the TEP shows correctly that this method can be applied to complex processes dealing with many variables.

5. Conclusions

In this paper, we have presented an approach for the fault detection and fault isolation of a multivariate process. This approach is based on a Bayesian network. We have combined previous work of control chart in a Bayesian network with some recent work of Li et al. [1]. The proposed approach allows us to isolate the variables responsible of a fault in a multivariate process. The method has been tested on a 5-variable system (a hot forming process) and on the benchmark problem of the TEP, demonstrating the performance of the method. This paper demonstrates the impact of taking into account causality in the detection and isolation steps.

Appendix A. Coefficient c demonstration

This appendix presents the demonstration of Eq. (7).

As for the \( T^2 \) control chart [11], we will fix a threshold (control limit CL for the control chart) on the a posteriori probabilities allowing to take decisions on the process: if, for a given observation \( x \), the a posteriori probability to be allocated to \( IC (P(\text{IC}|x)) \) is greater than the a priori probability to be allocated to IC \( P(\text{IC}) \), then this observation is allocated to IC. This rule can be rewritten as: \( x \in IC \) if \( P(\text{IC}|x) > P(\text{IC}) \), or equivalently \( x \in OC \) if \( P(\text{OC}|x) < P(\text{OC}) \). The objective of the following developments is to define \( c \) in order to obtain the equivalence between the Bayesian network and the multivariate \( T^2 \) control chart.

We want to keep the following decision rule:

\[ x \in IC \text{ if } T^2 < CL \]  
(A.1)

with this decision rule:

\[ x \in IC \text{ if } P(\text{IC}|x) > P(\text{IC}) \]  
(A.2)

We develop the second decision rule:

\[
P(\text{IC}|x) > P(\text{IC}) \]
\[ P(\text{IC}) > P(\text{IC}|x) + P(\text{OC}|x) \]
\[ P(\text{IC}|x) > P(\text{IC})P(\text{IC}|x) + P(\text{IC})P(\text{OC}|x) \]
\[ P(\text{IC}|x) > P(\text{IC})P(\text{IC}|x) + P(\text{OC})P(\text{OC}|x) \]
\[ P(\text{IC}|x) > P(\text{IC})P(\text{OC}|x) \]
\[ P(\text{IC}|x) > P(\text{IC}) \]
\[ P(\text{IC}|x) > P(\text{IC})P(\text{OC}|x) \]
\[ P(\text{IC}|x) > P(\text{IC})P(\text{OC}|x) \]

But, the Bayes law gives:

\[ P(\text{IC}|x) = \frac{P(\text{IC})P(\text{IC}|x)}{P(x)} \]  
(A.3)

and

\[ P(\text{OC}|x) = \frac{P(\text{OC})P(\text{OC}|x)}{P(x)} \]  
(A.4)

So, we obtain:

\[
\frac{P(\text{IC})P(\text{IC}|x)}{P(x)} > \left( \frac{P(\text{IC})}{P(\text{OC})} \right) \frac{P(\text{IC})P(\text{IC}|x)}{P(x)} \]
\[
\frac{P(\text{IC})P(\text{IC}|x)}{P(x)} > \left( \frac{P(\text{IC})}{P(\text{OC})} \right) P(\text{IC}|x) \]  
(A.5)

\[ P(\text{IC}|x) > P(\text{IC}) \]

In a Discriminant Analysis with \( k \) classes \( C_k \), the conditional probabilities are computed with the following Eq. (A.6), where \( \phi \) represents the probability density function of the multivariate Gaussian distribution of the class:

\[
P(x|C_k) = \frac{\phi(x|C_k)}{\sum_{k=1}^{k} P(C_k) \phi(x|C_k)} \]  
(A.6)

So, Eq. (A.5) can be written as:

\[ \phi(x|IC) > \phi(x|OC) \]  
(A.7)

We recall that the probability density function of a multivariate Gaussian distribution of dimension \( p \), of parameters \( \mu \) and \( \Sigma \), of an observation \( x \) is given by:

\[
\phi(x) = \frac{e^{-(1/2)(x-\mu)^T \Sigma^{-1}(x-\mu)}}{(2\pi)^{p/2} |\Sigma|^{1/2}} \]  
(A.8)

If the law parameters are \( \mu \) and \( c \times \Sigma \), then the density function becomes:

\[
\phi(x) = \frac{e^{-(1/2)(x-\mu)^T \Sigma^{-1}(x-\mu)}}{(2\pi)^{p/2} |\Sigma|^{1/2}c^{p/2}} \]  
(A.9)

In identifying the expression \( (x-\mu)^T \Sigma^{-1}(x-\mu) \) as the \( T^2 \) of the observation \( x \), we can write:

\[
\phi(x|IC) > \phi(x|OC) \]
\[
e^{-\frac{1}{2}T^2} > \frac{e^{-\frac{1}{2}T^2/2}}{(2\pi)^{p/2} |\Sigma|^{1/2}c^{p/2}} \]
\[
e^{-\frac{1}{2}T^2} > \frac{e^{-\frac{1}{2}T^2/2}}{(2\pi)^{p/2} |\Sigma|^{1/2}c^{p/2}} \]
\[
T^2 < \frac{T^2}{c} + \frac{p \ln(c)}{2} \]
\[
T^2 < \frac{p \ln(c)}{1 - (1/c)} \]  
(A.10)

However, we search the value(s) of \( c \) allowing the equivalence with the control chart decision rule: \( x \in IC \) if \( T^2 < CL \). So, we obtain the following equation for \( c \):

\[
\frac{p \ln(c)}{1 - (1/c)} = IC \]  
(A.11)

Or, equivalently:

\[ 1 - c + \frac{pc}{IC} \ln(c) = 0. \]  
(A.12)

References